

## Written Exam for the M.Sc. in Economics, Winter 2019-20

### Advanced Macroeconomics: Structural Vector Autoregressive Analysis: Solution

#### About the exam

The topic for this project examination is the money demand function stating that the demand for money is a function of interest rates and output. The standard long-run money demand function can be written as

$$m = \beta_0 + \beta_1 y + \beta_2 (r^S - r^O) \quad (1)$$

where  $m$  is the natural logarithm of real money balance ( $M/P$ ),  $y$  is the natural logarithm of real GDP,  $r^S$  is the short-run interest rate and  $r^O$  is the “own rate” of return of components in the money stock. The spread ( $r^S - r^O$ ) represents the opportunity cost of holding money balances.

There is a large literature estimating this money demand function and studies of the effects of monetary policy. One puzzle that emerges in this literature is that restrictive monetary policy (an increase in the money market rate) is associated with increased prices. There have been many attempts in the literature to understand and/or explain this result. The present assignment provides an additional approach to investigate this puzzle.

Approximate the spread using the money market rate, i.e.,  $r = (r^S - r^O)$ . We can then rewrite the money demand function above as the following long-run relationship

$$m = \beta_0 + \beta_1 y + \beta_2 r. \quad (2)$$

This implies that if the variables in the equation above are  $I(1)$ -processes, then the three variables must cointegrate.

Regarding the data for the exam paper, please note the following:

- All assignments are based on different data sets. You should use the data set (monthly data covering the period 1979:02-2003:12) located in the MATLAB file 1234.mat, where 1234 is your exam number. This MATLAB file contains the data ( $y$ ), the dates (dates) and the name of the variables (names). You can load this file into MATLAB directly using 'load 1234.mat'. In case you cannot find your exam number, you can use the 1000.mat file.
- To avoid that some data sets are more difficult to handle than others, the data sets are artificial (simulated from a known data generating process), and they behave, as close as possible, like actual data.

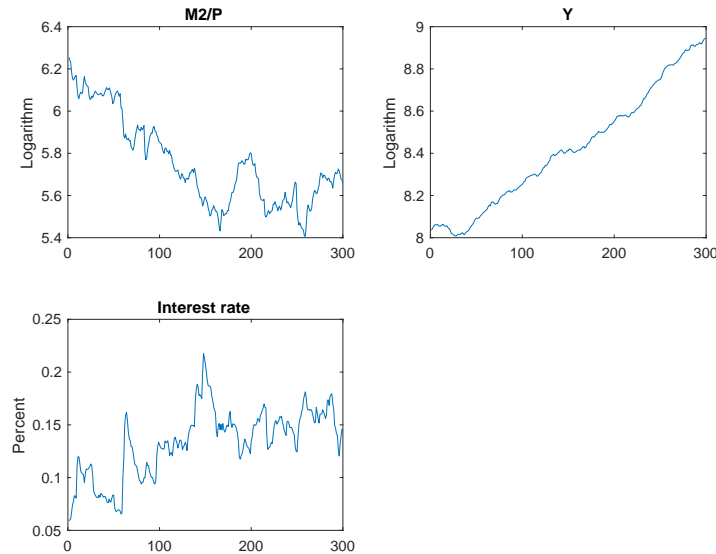
The proposed solution below is based on the dataset 1000.mat

1. The data is already in natural logarithms (real money balance and real GDP are in logs whereas the money market rate is in percent). Plot the data and perform graphical analysis in order to assess the degree of integration of all three variables.

#### Answer:

As is evident from this graph, there seems to be linear trends in the data. Output  $Y$  is increasing over time whereas  $M/P$  is decreasing. It may be that there is a linear trend in the interest rate spread also. However, it is an empirical question whether a linear trend is needed in the cointegration vector.

Figure 1: Money demand data.



## The Vector Error Correction Model

Suppose that the three variables in the money demand function are either  $I(1)$  or  $I(0)$  and that the underlying data generating process is a trivariate Vector Autoregressive (VAR) model,

$$y_t = \nu + A_1 y_{t-1} + \dots + A_p y_{t-p} + u_t \quad (3)$$

where  $y_t = [m \ y \ r]'$ ,  $p$  is the lag length,  $\nu$  is a constant term and  $u_t$  is a trivariate zero mean white noise process with covariance matrix  $\Sigma_u$  such that  $u_t \sim (0, \Sigma_u)$ . Then we can rewrite the VAR model as the following Vector Error Correction (VEC) model

$$\Delta y_t = \nu + \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_{p-1} \Delta y_{t-p+1} + u_t \quad (4)$$

where

$$\Pi = -(I_3 - A_1 - \dots - A_p)$$

and

$$\Gamma_i = -(A_{i+1} + \dots + A_p) \quad \text{for } i = 1, \dots, p-1.$$

The rank of  $\Pi$  is equal to the number of cointegration vectors  $r$  and can be decomposed as a product of two  $3 \times r$  matrices of full rank,  $\Pi = \alpha \beta'$  where  $\alpha$  is the  $3 \times r$  adjustment coefficients and  $\beta$  is the  $3 \times r$  cointegration vectors.

2. Formulate a well-specified VEC model for  $y_t$  similar to the VEC model above. Explain your workflow and how you argue for your choice of the number of autoregressive lags in the VEC model (and in its associate VAR model).

**Answer:**

There are several different approaches that can be used to determine the number of lags in the underlying VAR model. All approaches are based on estimates of a VAR in levels with a constant term. Three approaches have been used during the course: lag length determination using information criteria, general-to-specific and specific-to-general sequences. It is irrelevant which one is used here. All approaches should lead to the same lag length. The first step in empirical VAR analysis is to determine the lag length in the underlying VAR. Regardless, it is important that the workflow is explained. Here we will apply information criteria (Akaike, Schwarz and Hannan-Quinn) and choose the lag length that minimizes these measures. The workflow should start with a maximum lag length and then we compute these criteria for each lag length  $p = 0, \dots, p_{\max}$  using the same number of observations for each lag length. The function `pfind.m` produces the following output assuming that  $p_{\max} = 36$  (but we only report the information criteria for lags less than or equal to 10). As is clear from this table, the lag length suggested by all criteria is equal to 2. The same result holds for all data sets and all approaches.

$p$	SIC	HQC	AIC
0	-14.976	-15.001	-15.017
1	-28.461	-28.558	-28.623
2	-28.771	-28.941	-29.056
3	-28.618	-28.861	-29.024
4	-28.461	-28.777	-28.989
5	-28.298	-28.687	-28.948
6	-28.122	-28.584	-28.894
7	-27.964	-28.499	-28.858
8	-27.815	-28.422	-28.83
9	-27.693	-28.374	-28.831
10	-27.549	-28.302	-28.808

Alternatives to using information criteria as outlined above is to use one of the following approaches:

- Top-down sequential testing (general-to-specific): The VAR( $p$ ) model is

$$y_t = \nu + A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + u_t$$

where  $\nu = A_0$ . Start with a maximum number of lags  $p_{\max}$  testing a sequence of null hypotheses:  $\mathbb{H}_0: A_{p_{\max}} = 0$  vs.  $\mathbb{H}_1: A_{p_{\max}} \neq 0$ ,  $\mathbb{H}_0: A_{p_{\max}-1} = 0$  vs.  $\mathbb{H}_1: A_{p_{\max}-1} \neq 0$ , ...,  $\mathbb{H}_0: A_1 = 0$  vs.  $\mathbb{H}_1: A_1 \neq 0$ . Process terminates when there is a rejection. Use Wald or LR tests.

- Bottom-up sequential testing (specific-to-general): Reverse the procedure above, start with  $p_{\min}$  testing for autocorrelation in the residuals (using for example a multivariate test). Add lags until there is no significant autocorrelation.

Answers using either of these two approaches should also be accepted and receive full points if correctly implemented and explained.

3. Test for multivariate autocorrelation, heteroscedasticity and normality. Does your model satisfy the underlying assumptions?

**Answer:** To verify that the model is well-specified when assuming that the lag length is equal to 2, we next test for autocorrelation and heteroscedasticity in the residuals and the null hypothesis that the residuals are normally distributed. We should use multivariate tests provided in the functions: `portman.m`, `march.m` and `multnorm.m`. First we need to re-estimate the VAR(2) model using `VARls.m`. We allow for a constant term but no linear trend. The argument used to exclude a linear trend is that the VAR in levels can be re-written as a VAR in first differences (and as a VEC model) by subtracting  $y_{t-1}$  from both sides of the levels VAR leaving the constant term and the residuals unaffected.

Start with the multivariate test for autocorrelation. We need to specify the horizon used when computing the test statistic. We start with horizon equal to 12. The `portman.m` function gives us the following result:

Portmanteau test Test	Statistic
-----	-----
Tested order:	12
Test statistic	86.681
p-value	0.5795
Adjusted test statistic	88.868
p-value	0.51395
degrees of freedom	90

indicating that we cannot reject the null hypothesis of no autocorrelation in the residuals.

Next, we test for multivariate ARCH using the `march.m` function. The results suggest that we, for number of lags = 6 cannot reject the null hypothesis that there are no ARCH effects in the residuals at the 5 percent level.

Tests for Multivariate ARCH	
Test	Doornik_Hendry
-----	-----
test statistic:	236.82
p-value	0.15793
degrees of freedom	216

Finally, we test for normality using the `multnorm.p` function.

Tests for non-normality Test	Doornik_Hansen	Lutkepohl
joint test statistic:	1602.6	1395
p-value	0	0
degrees of freedom	6	6
Skewness only	200.28	178.51
p-value	0	0
kurtosis only	1402.3	1216.5
p-value	0	0

The joint test statistic suggests a rejection of the null that the residuals are normally distributed. Individual tests suggests the presence of kurtosis but no skewness at the 5 percent level. This result is common in most data sets and should not pose a serious problem for the following analysis.

Overall these test statistics suggest that the model with  $p = 2$  is fairly well-specified. Adding one lag does not solve any problems detected by the normality tests. A good answer provide a discussion and should end with the conclusion that  $p = 2$  seems reasonable.

## Testing for cointegration

4. For your preferred model, proceed by testing for cointegration using the MATLAB function `jcitest`. Explain your approach and how you proceed to find the number of cointegration vectors in the system, that is the rank  $r$ . Do you use different sources of information when determining the rank? If so, explain how you arrive at your decision.

**Answer:** Assuming that  $p = 2$  in the underlying VAR implies that there is only 1 lag in the VEC model. Remember that the MATLAB function `jcitest` requires the user to specify  $p - 1$  instead of  $p$ , a good answer must explicitly mention the number of first difference lags used in the tests. Using the `jcitest` function we obtain:

r	h	stat	cValue	pValue	eigVal
0	1	41.7123	29.7976	0.0018	0.1030
1	0	9.3206	15.4948	0.3838	0.0290
2	0	0.5413	3.8415	0.6186	0.0018

Starting with the hypothesis that  $r = 0$  we find that the null hypothesis that  $r = 0$  is rejected at the 1 percent level. Increasing the rank we find that we cannot reject the null that  $r = 1$ . The data, therefore, suggest the presence of 1 cointegration vector.

This result holds for any lag length  $\leq 6$ . So if it was decided to increase the number of lags in the underlying VAR above to handle the autocorrelation and ARCH findings, the Johansen test still suggest the presence of 1 cointegration vector.

We can use the estimated eigenvalues as an alternative source of information when determining the rank. As is clearly illustrated in the table above, the eigenvalue falls

substantially when increasing the rank. Our conclusion that the rank is equal to 1 is clearly supported.

A good answer must include a description of the approach, the so called Pantula principle, where we start by considering the null hypothesis that the rank is 0 and then we test the null that the rank is 1 and so on. A sensitivity analysis of the unimportance of the number of lags should also be included. As mentioned above, the answer must explicitly state how many lags there are in the VEC model estimation.

5. Discuss and argue for the preferred way to include deterministic components in the model. Perform a test of the null hypothesis that there is no linear trend in the cointegration vectors.

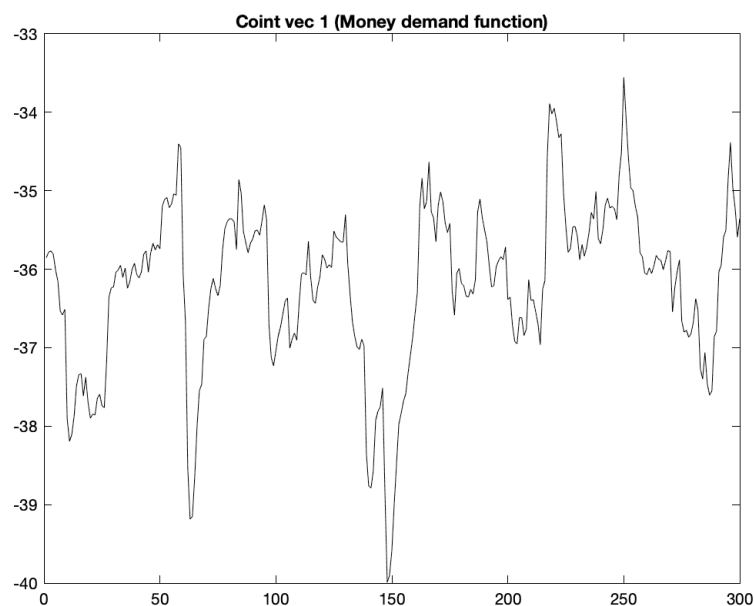
**Answer:** The Johansen test computed above rests on the assumption that there is no linear trend in the cointegration vector. We can distinguish between five different cases depending on assumptions about deterministic components. In our case, it is evident that we need a constant term in the VEC model. The question is whether we also should add a linear trend, either a linear trend in the cointegration vector but no quadratic trend in the data or allowing for a quadratic trend in the data.

We can test the null hypothesis that there is a linear trend in the cointegration vector using the `jcontest` function in MATLAB. The MATLAB code to be used is:

```
[h,pValue] = jcontest(y,1,'Bcon',[0 0 0 1]','model ','H*','lags',p-1);
```

Executing this command we find a test statistic of 0.4331 with a p-value of 0.5105, thus indicating a non-rejection of the null hypothesis that the linear trend can be excluded from the cointegration space. This holds for all data sets.

A good answer should include a formal test of this null hypothesis and a motivation based on the graph of the cointegration vector where it can be seen that there is no apparent linear trend. Below is a graph of the unrestricted cointegration vector.



6. Impose your preferred rank and test hypotheses on the cointegration space using the MATLAB function `jcontest`. Start with tests for exclusion, stationarity and weak exogeneity. Explain the meanings of these tests.

**Answer:** Stationarity test, exclusion tests and tests for specific cointegration vectors are related. If one variable can be excluded from the cointegration space, then we expect to reject cointegration vectors where this variable is included. If a variable is stationary, then this variable forms a cointegration vector where all other variables are excluded. Weak exogeneity implies that there is no error correction in the associated equation. In this case, that equation can be excluded from the VEC model. The test results should be consistent, but this may not always be the case in practice.

Using this data set we find that we always reject the null hypothesis that the variables are stationary. Tests for weak exogeneity suggest that we can reject the null that real money balance and the interest rate spread can be excluded from the cointegration vector whereas real money balance is weakly exogenous. A good answer should include the test statistics and an explanation of the tests. Results may differ across the data sets.

7. Estimate the parameters in the cointegration vectors and interpret these in light of the money demand function above. Do you find plausible values of the parameters in the cointegration vectors? Test null hypotheses using the MATLAB function `jcontest`. Explain how this test relates to the exclusion and stationarity tests.

**Answer:** A good answer must include an estimate of the unrestricted (but normalized) cointegration vector. The unrestricted estimated money demand function (the cointegration vector) is:

$$M/P = 0.2152 \times Y - 8.5361 \times r$$

We can compare this vector to, for example, the papers included in the curriculum. We find that the estimates above are clearly in the range of common estimates in the literature using different data sets. Real money balance is often related to output in a one-by-one relation but here we find a much smaller parameter estimate (but not uncommon in the literature) and the interest rate elasticity parameter conforms to standard estimates. It seems that we can interpret the cointegration vector as a money demand function (signs are correct and parameter estimates close to standard estimates). A good answer also includes a formal test of the null hypothesis that the parameter associated with output and the real money balance are equal but of opposite sign. An estimate of the restricted cointegration vector must be included.

8. Split the sample into two equal sized sub-samples and perform tests for exclusion, stationarity and weak exogeneity. Comment on the importance of the sample length for these tests.

**Answer:** Here we will find that the information criteria still suggest that  $p = 2$ , Johansen tests are often but not always consistent. For this data set we find that the rank may be equal to 2 in the first part of the sample but equal to one in the latter part of

the sample. Tests for stationarity, exclusion and weak exogeneity also change. This illustrates that the sample length is of importance for results but this fact must be viewed in light of the power of the tests. Using few observations may bias the test statistics and it is of importance to test for stability. We know that the Johansen test and tests on the cointegration vectors in particular are biased in small samples. For our sample sizes (and of course the simulated data) we only find that tests on the cointegration vector change when reducing the number of observations. The cointegration vector, given lag length equal to 2 and no linear trend in the cointegration vector, may not always be consistent with the full sample estimates.

9. Impose  $r = 1$  and re-estimate the VEC model using the full sample and using your preferred lag length found above. Comment on the driving forces of the data in this model.

**Answer:** We can estimate the VEC model with known cointegration vector using either LS or ML. In our model we have rank equal to 1 and 3 variables. This implies that there are 2 common trends driving the 3 variables. These two common trends determine the long-run behavior of the three time series, that is, they have long-run effects on all variables. Since we have one cointegration vector, we have 1 transitory shock affecting the variables only in the short-run.

A good answer includes both estimates of the VEC model and a short discussion about the driving forces as above.

## Identification of structural model

10. Suggest identification schemes including names of the three structural shocks in the VAR/VEC system using first a Cholesky decomposition and second long-run identification. If you cannot provide names for these shocks, try to explain how they affect the data under the maintained assumptions.

**Answer:** First of all we need to decide upon the ordering of the data. Monetary policy (the interest rate spread) should respond to all three shocks in the VAR system and output may respond with a lag to monetary policy. Then, it is a matter of choice whether real money balance should respond immediately to output shocks or not. In the curriculum, we have discussed a number of papers estimating the effects of monetary policy during the course (Sims, Eichenbaum and Uhlig). Common in these papers is that the interest rate spread is the last variable in the system (given that we use a lower triangular Cholesky decomposition). Sims usually put the money stock and prices above output whereas Eichenbaum and Uhlig often put output first in the time series vector. We follow Uhlig and Eichenbaum and define the time series vector as  $y_t = (y \ m \ r)'$ .

This ordering is also preferable when taking cointegration into account. In this case we may want to put output first (arguing that the first permanent shock is a productivity shock) and assume that the other two shocks do not affect output in the long-run. This is standard in the literature. Then we may interpret the second shock as a real money balance (or demand) shock having no long-run effect on output.



A good answer must include a discussion along the lines above.

11. Write down the reduced form and structural form Common Trends model consistent with the VEC model. Show how these two representations are related.

**Answer:** The underlying VAR model can be rewritten as the following VEC model

$$\Delta y_t = \alpha \beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_{p-1} \Delta y_{t-p+1} + u_t$$

where  $\alpha$  is the adjustment coefficients and  $\beta$  is the cointegration vector. The VEC model can then be rewritten as a Common Trends model

$$y_t = \Xi \sum_{i=1}^t u_i + \Xi^*(L)u_t + y_0^*$$

where

$$\Xi = \beta_{\perp} \left[ \alpha'_{\perp} \left( I_K - \sum_{i=1}^{p-1} \Gamma_i \right) \beta_{\perp} \right]^{-1} \alpha'_{\perp}$$

where  $\alpha_{\perp}$  and  $\beta_{\perp}$  are orthogonal complements to  $\alpha$  and  $\beta$  respectively. The structural Common Trends model can be written as

$$y_t = \Xi \sum_{i=1}^t B_0^{-1} w_i + \Xi^*(L)B_0^{-1}w_t + y_0^*$$

$$y_t = \underbrace{\Xi B_0^{-1}}_{\Upsilon} \sum_{i=1}^t w_i + \Xi^*(L)B_0^{-1}w_t + y_0^* = \Upsilon \sum_{i=1}^t w_i + \Xi^*(L)B_0^{-1}w_t + y_0^*$$

where  $\Upsilon$  is the matrix of long-run multipliers, it measures the long-run effect of the common trends (or the permanent shocks). Note that the long-run effects of the stationary part  $\Xi^*(L)B_0^{-1}w_t$  goes to zero as  $j \rightarrow \infty$ . The rank of  $\Upsilon$  is the same as the rank of  $\Xi$ , i.e., rank  $K - r$ .

Long-run restrictions can be imposed directly on  $\Upsilon$ , if the long-run effect of a shock is zero on all variables, then the corresponding column of  $\Upsilon$  is restricted to zero. To identify the transitory shocks we impose restrictions on  $B_0^{-1}$ , in particular, we impose restrictions on the last  $r$  columns of this matrix.

The interpretation of the permanent and transitory shocks in the current setting is discussed above. We associate the two permanent shocks with productivity and real money balance shocks. These shocks have permanent effects on at least one of the three variables in the system. To just identify these two shocks we need to introduce one restriction. We then have one transitory shock, the monetary policy shock. This shock does not have any long-run effect on the three variables. Since there is only one transitory shock, it is identified (since the permanent and transitory shocks can be identified independently).

A good answer must include a discussion of possible shocks affecting the VAR/VEC system. The arguments above may not be the only available option, but it is essential that the answer includes a motivation and main arguments based on economic model or intuition.

12. What is the consequence for the long-run multiplier if we assume that the long-run money demand function is a cointegration vector?

**Answer:** The long-run multiplier  $\Upsilon$  is a  $3 \times 3$  matrix where the third column is zero since the long-run effects of the transitory shock on all variables is zero. To identify the permanent shocks we use the condition that  $\beta' \Upsilon = 0$ .

13. Outline how the MATLAB solver can be used to impose long-run restrictions.

**Answer:** The following code implements the restriction:

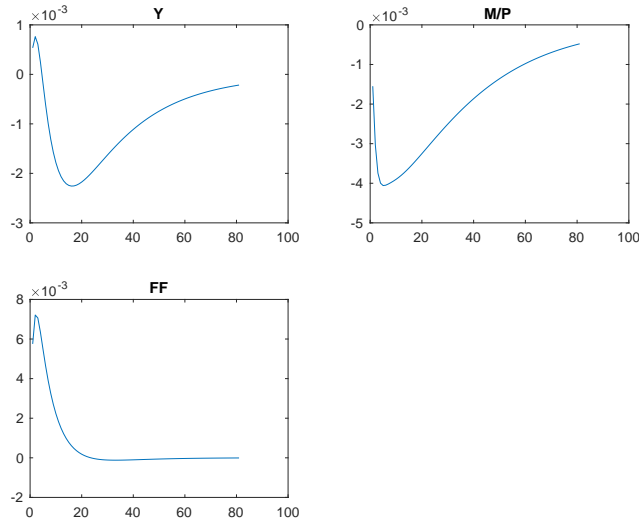
```
% restrictions.M
% Normalization: SIGMA_w=I
function q=restrictions(B0inv)
global GAMMA SIGMA alpha beta alpha_perp beta_perp Xi p
K=size(B0inv,1);
THETA1=Xi*B0inv;
F=vec(B0inv*B0inv'-SIGMA(1:K,1:K));
% Long run and short run restrictions
q=[F; THETA1(2,1); THETA1(1,3); THETA1(2,3); THETA1(3,3)];
q'+1;
```

where the notation is standard.

## Impulse responses and forecast error variances

14. Estimate a VAR model (using your preferred number of lags found previously) with all variables in log-levels and use this model together with a Cholesky decomposition to compute impulse response functions. Discuss and explain how you order the time series vector, i.e., explain and motivate the identifying restrictions you use. Show the impulse responses in a graph and interpret your results. Comment on the price puzzle. Is it still there in the model or not? If not, try to provide an explanation. No confidence bands are required.

**Answer:** Use the ordering of the variables above and estimate a VAR in levels with a linear trend and using a lower triangular Cholesky decomposition we can compute the impulse response functions. We have decided to include a linear trend since the graphs of the data suggest that, at least, two of the time series were trending. Using the ordering suggested above and a lower triangular Cholesky decomposition we find the following impulse responses of all three variables to a monetary policy shock. All data sets should generate the same patterns as below but point estimates could differ.



A positive monetary policy shock initially increases output but after a few quarters, output falls. The effect on real money balance is negative. A higher interest rate should be associated with a reduction in the money supply and if the price level falls with a lag and slower than the rate by which the money supply falls, then real money balance should fall as implied in the graph above. If money supply is constant, real money balance should increase since the higher interest rate should lead to a lower price level. This alternative interpretation is not consistent with the impulse response above. The prize puzzle often found in the literature implies that a higher interest rate is associated with higher price level. Holding money supply constant, a higher price level would also reduce real money balance. This case is also consistent with the impulse response above. It is therefore unclear whether we have solved the price puzzle or not. However, a likely scenario is that the higher interest rate reduces the money supply and that the price level responds to the higher interest rate with a time lag. This is consistent with the impulse response where we obtain a large fall in the real money balance in the short run and then as the price level falls, real money balance increases slowly over time. A good answer must include interpretation and discussion.

15. Instead of using a standard VAR and recursive identification we can identify the model using a combination of short-run and long-run identifying restrictions. Use the MATLAB solver to identify the structural VEC model estimated previously. Check that the solver provides a valid identification and compute the variance-covariance matrix of the identified structural shocks. Please, provide the MATLAB code you are using to identify the shocks in the appendix. It must include a description of the null restrictions you impose.

If you fail computing the  $B_0^{-1}$  matrix using the MATLAB solver, please use the ident.p file. This file works as a standard m-file but the coding is concealed and there is no way to convert the p-file into an m-file. Note that the ident.p file is set up to use the closed form solution to compute the  $B_0^{-1}$  matrix using a generic identification based on

estimates from the VEC model. To use this function, you need to add the following code to your MATLAB m-file and you need to set the rank equal to 1. You can use any number of lags. Note: Make sure that you don't have any ident.m files in the same folder. The same function can be used in a bootstrap. Note also that it will be impossible to interpret the impulse response function and the variance decomposition using economic theory.

```

% Use generic identification
% Input:
% alpha is the 3 x 1 adjustment coefficient matrix
% beta is the 1 x 3 cointegration vector [beta1 beta2 beta3]
% Gamma = [ Gamma(1) Gamma(2) ... Gamma(p-1)] coefficient matrix
% sigmahat = Sigma_u (the residual covariance matrix)
% K = 3
% p = number of lags in underlying VAR
% r = 1
%
% Output:
% inverse of B0 matrix
[invB0]=ident(alpha,beta,Gamma,sigmahat,K,p,r)

```

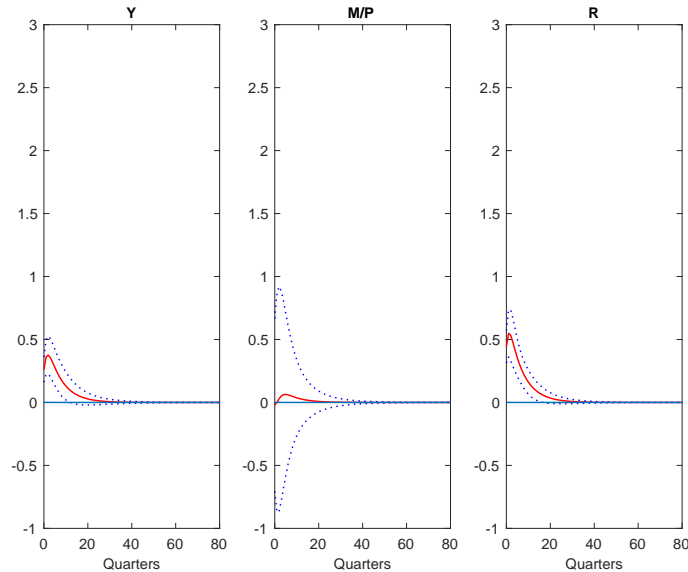
**Answer:** The MATLAB code provided in the answer to question 13 above can be used to estimate the structural model. Note that the ident.p file implements another identification scheme, it may not be possible to provide an interpretation of the two permanent shocks in this case. However, using this generic identification allows the user to estimate a structural model. In either case, it is necessary to show that the solver has found a proper solution. Apart from providing an estimate of the  $B_0^{-1}$  matrix, it is necessary to show the following matrices:  $B_0^{-1}(B_0^{-1})' - \Sigma_u$  which should be equal to zero;  $\Upsilon$  which should include the zero restriction imposed on one element in the first two columns;  $B_0 \Sigma_u B_0'$  which is the variance-covariance matrix of the structural shocks  $w_t$  which should be an identity matrix.

A good answer implements the proposed identification correctly and should also provide statistics verifying that the solution is valid using the statistics mentioned in the previous paragraph.

16. Estimate the structural VAR model and compute impulse response functions (with bootstrap confidence bands using the delta method, i.e., the standard residual based recursive design bootstrap with intervals based on bootstrap standard errors) and variance decompositions (with bootstrap standard errors using Efron's percentile intervals). You can show forecast error variance decompositions in either a table or in a graph. Interpret your results.

**Answer:** The impulse responses to a positive shock to the interest rate spread is shown in the figure below together with the 95% confidence bands generated using standard residual based bootstrap with 500 trials. The responses are quite different from what we obtained using the Cholesky decomposition. An increase in the interest rate spread is now associated with an increase in output, which is clearly against convention, while

Figure 2: Impulse responses of all variables to a positive monetary policy shock.



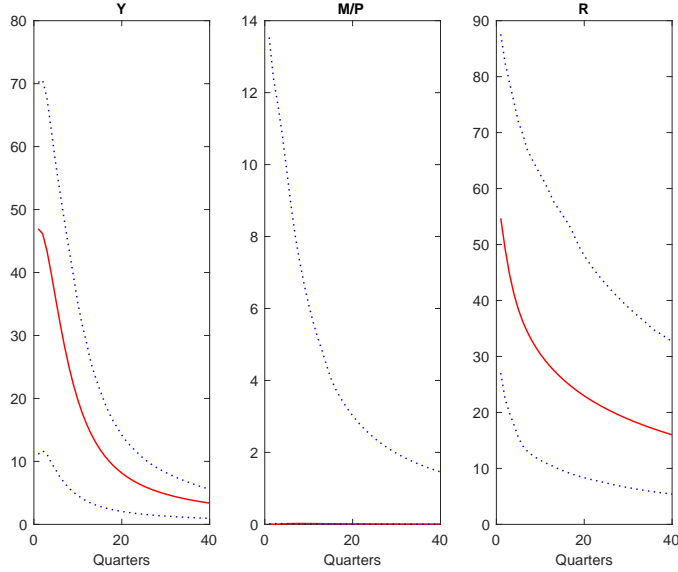
the response of real money balance is insignificant. This last result is consistent with the view that both money supply and the price level responds in similar ways (money supply falls and the price level falls). Looking more closely at the responses of the variables to the other two shocks we also find inconsistent effects. It seems as if our identification is incorrect or incomplete. The cointegration vector used here is consistent with a standard money demand function and this vector is used to identify the two groups of shocks. But, it may be that the cointegration tests understate the number of vectors (all variables may be stationary around linear trends). Or, the cointegration tests may overstate the number of vectors such that all variables are non-stationary. To further analyze this, one could estimate a VAR in levels but with linear trends and check the stability of this model. This is not required, though. A good answer should include an interpretation of the results and an evaluation whether the findings are consistent with standard theory. Extra credit should be given for answers that also include estimates and stability tests of a standard VAR model with data in levels and a linear trend (or maybe also a quadratic trend).

The forecast error variance also shown below suggest that monetary policy shocks explain sizable portions of output at least in the short-run. Confidence bands are constructed using Efron's bootstrap method.

17. Show a plot of the accumulated permanent shock(s) and the three variables. Discuss the results. Is it possible to draw conclusions regarding the driving forces of the variables?

**Answer:** The graph below shows the accumulated structural shocks  $w_t$ . Again, these shocks do not resemble the time series shown above and therefore again questions the identification. The accumulated permanent shocks represent the non-stationary part of the time series and a weighted average (through  $\Upsilon$ ) determines the permanent part of

Figure 3: Forecast error variance decomposition, positive monetary policy shock.



each time series. A good answer explains how the graph is constructed and also adds a short discussion.

## Extensions

18. An alternative to using the MATLAB solver to compute the  $B_0^{-1}$  matrix is to use the approach suggested by Warne (1993). Outline this approach and show how the Warne approach can be used to identify the structural shocks in your preferred VEC model. Discuss both the identification of permanent and transitory shocks.

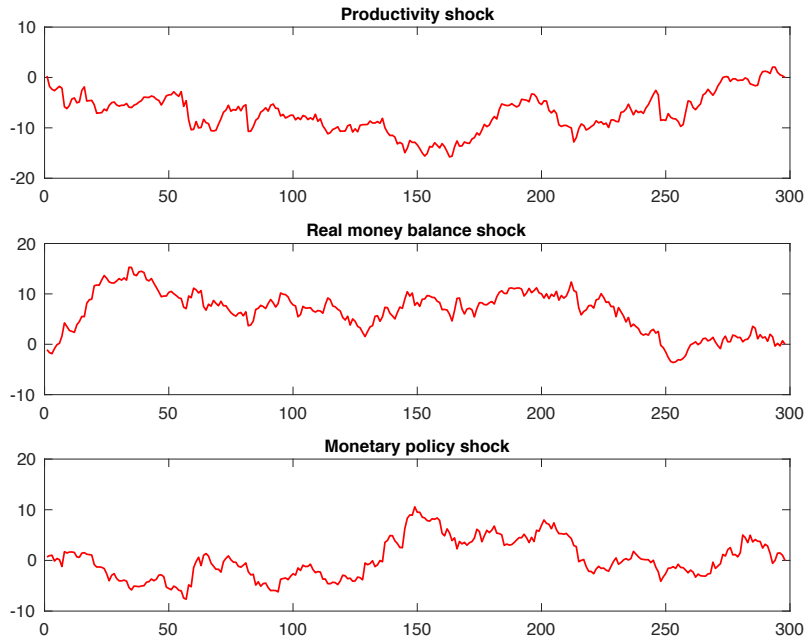
**Answer:** Following Warne we first define the matrix  $\Upsilon_0$  to identify the the two permanent shocks and then we define the  $U$  matrix. We have one restriction that we need to impose to identify the two permanent shocks. Assume that real money balance shocks cannot affect output in the long-run, this corresponds to a zero restriction in the first row and second column of  $\Upsilon$ . To find an  $\Upsilon_0$  matrix leading us to the  $\Upsilon$  matrix we first make use of the restriction that  $\beta'\Upsilon_0 = 0$ . This implies that

$$\Upsilon_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1/\beta_{1,3} & -\beta_{1,2}/\beta_{1,3} \end{bmatrix}.$$

where we have normalized  $\beta$  on the first element. From Warne we know that  $\Upsilon = \Upsilon_0\pi$  where  $\pi$  is a lower triangular matrix. The matrix  $\Upsilon$  is then given by

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1/\beta_{1,3} & -\beta_{1,2}/\beta_{1,3} \end{bmatrix} \begin{bmatrix} \pi_{11} & 0 \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} \pi_{11} & 0 \\ \pi_{21} & \pi_{22} \\ -\pi_{11}/\beta_{1,3} - (\beta_{1,2}\pi_{21})/\beta_{1,3} & -\beta_{1,2}\pi_{22}/\beta_{1,3} \end{bmatrix}$$

Figure 4: Accumulated structural shocks.



where we see that the identifying restriction that the second permanent shock has no long-run effect on the first variable is imposed.

Since we have only 1 transitory shock,  $U = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$ .

It is now straightforward to code this. For this ordering of the variables we can verify that the  $B_0^{-1}$  matrix found using the Warne method is equal to the one found by the solver. It could be that we need to switch signs on one or two columns of the solver solution, but the absolute values are identical.

19. Code this identification and compute the implied  $B_0^{-1}$  matrix and show that it is identical to the one found by the solver. Please, provide the code you are using in the appendix.

**Answer:** It is enough here to show that the  $B_0^{-1}$  matrix obtained using the Warne approach is equal to the solution obtained using the MATLAB solver. The answer must include the code used to compute the closed form solution. The code used could look like (the code is adapted from the KLversusWarne.m file). The answer must include the MATLAB code and the definition of the matrices above.

```
% Warne identification
Upsilon0 = [1 0 -1/beta(1,3);0 1 -beta(1,2)/beta(1,3)]';
MHLP=inv(Upsilon0'*Upsilon0)*Upsilon0'*Xi;
pipit=MHLP*SIGMA*MHLP';
pimat=chol(pipit)';
Upsilon=Upsilon0*pimat;
Fk=inv(Upsilon'*Upsilon)*Upsilon'*Xi;
display(Fk,'Fk matrix');
```

```

Umat=zeros(1,3);
i=1;
while i<=r;
Umat(i,K-i+1)=1;
i=i+1;
end;
% Check that identification of transitory shocks is valid
if det(Umat*alpha)==0
display('Identification of transitory shock is invalid');
else
display('Identification of transitory shocks is valid');
end
xi=alpha*inv(Umat*alpha);
i=1;
while i<=K;
j=1;
while j<=r;
if abs(xi(i,j))<=1E-12;
xi(i,j)=0;
else
end
j=j+1;
end
i=i+1;
end
qr=chol(xi'*inv(SIGMA)*xi)';
Fr=inv(qr)*xi'*inv(SIGMA);
display(Fr,'Fr matrix');
invB0 = inv([Fk;Fr]);
invB0 = [-invB0(:,1) invB0(:,2:3)];
display(invB0,'B0^1 matrix');
display(-Xi*invB0,'(3) C(1)*B0^1 should be Upsilon zeros(K,r)');
display(inv(invB0)*SIGMA*inv(invB0)', '(4) Covariance matrix of structural shocks
w_t should be I_K');
display(inv(qr)*xi'*inv(SIGMA)*xi*inv(qr)', 'Should be diagonal');

```